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SIMULATION OF VERTICAL TRANSPORT OF HEAT AND SOLID PARTICLES IN FLUIDIZED-BED APPARATUS

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An equation for simulating nonstationary vertical transport of heat and solid particles in nonhomogeneous fluidized beds is proposed.

The trends in the transport of solid particles and the related heat transport* throughout the fluidized bed space strongly affect the operating efficiency of apparatus based on fluidization techniques. Therefore, the mixing of particles and internal heat transfer in such a system invariably engage the interest of researchers [1-4].

Until recently, the most commonly accepted vertical mixing scheme was based on the classical diffusion model, which describes the process by means of a single parameter – the coefficients of vertical diffusion (dispersion) of particles [1]. However, this model cannot describe the experimentally observed nonstationary mixing curves [5].

It was proposed in [6], probably for the first time, to describe the process by a hyperbolic diffusion equation that would take into account the finiteness of the particle velocities. A system of two hyperbolic first-order equations was used earlier in [7] for describing the vertical mixing of the solid phase. This system was based on the assumption that the transport of particles throughout the bed was purely convective (circulatory) in character: upward in the bubble trails and downward in the rest of the emulsion phase. In this, the particle velocities in both phases were, naturally, limited. The necessity and importance of taking into account the finiteness of the velocity of particles was shown in [5, 8] by direct comparison between the experimental mixing curves and those calculated by means of hyperbolic equations [6, 7]. Using the results of an analysis of the fluidization process based on methods of the thermodynamics of irreversible processes, Liu and Gidaspow have derived [3] a hyperbolic equation of diffusion to describe vertical solid phase transport. It has been suggested in [4] to use three first-order hyperbolic equations to describe vertical mixing of particles in a bed slowed down by a bunch of pipes. An additional equation (in comparison with the system given in [7]) describes the downward core motion of particles at the wall. Analysis shows that none of the above models comprises all the basic characteristics of the mixing process (see below).

* It is admissible to assume that the heat transfer is due entirely to the motion of particles in nonhomogeneous fluidized beds because of the large difference between the volumetric specific heat values of the gas and the particles. The transport of heat and the transport of disperse material and therefore characterized by the same trends, so that, for brevity, we shall subsequently make no special distinction between these processes and use only the term "mixing" (diffusion of particles) or "thermal conductivity" of the bed.

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The parameters of these models, as a rule, do not reflect all of the actual physical processes, which makes it impossible to describe the diffusion of particles in a wide range of experimental conditions. The two-concentration model of mixing which is presented in [9] takes into account, besides the finiteness of the particle velocity, also other important characteristics of the process, which are not considered all together in other schemes: convective particle transport, exchange between the ascending and descending particle flows, and turbulent particle diffusion in the descending emulsion phase, caused by passage through a layer of gas bubbles. The distinguishing feature of this model is that it is suitable for describing both horizontal and vertical mixing, as it establishes a close mutual relationship between them. The present study is a continuation of [9], and its aim is to determine the possibility of describing vertical particle mixing and heat transport in nonhomogeneous fluidized beds by means of a second-order hyperbolic equation.

The following system of equations is used in [9] to describe vertical mixing of particles in a nonhomogeneous fluidized bed:

$$A\frac{\partial c_1}{\partial t}+u\frac{\partial c_1}{\partial x}+A\tau^*\frac{\partial^2 c_1}{\partial t^2}+2u\tau^*\frac{\partial^2 c_1}{\partial t\partial x}=\left(AD_{xx}-\frac{\tau^* u^2}{A}\right)\frac{\partial^2 c_1}{\partial x^2}+$$
$$+\beta(c_2-c_1)\left(1+\tau^*\frac{\partial}{\partial t}+\tau^* u_1\frac{\partial}{\partial x}\right);\quad B\frac{\partial c_2}{\partial t}-u\frac{\partial c_2}{\partial x}=\beta(c_1-c_2). \tag{1}$$

We can readily derive from (1) an equation for calculating the vertical profile of the mean particle density $c = (Ac_1 + Bc_2)/(A + B)$:

$$\frac{\partial c}{\partial t}+\left[\tau^*+\frac{AB}{\beta(A+B)}\right]\frac{\partial^2 c}{\partial t^2}+\left[\tau^* u_1+\frac{(B-A)u}{\beta(A+B)}\right]\frac{\partial^2 c}{\partial t\partial x}-\left[D_V+\frac{u^2}{\beta(A+B)}\right]\frac{\partial^2 c}{\partial x^2}+\frac{1}{\beta}\left[\frac{\tau^* u^2(B-2A)}{A(A+B)}-BD_V\right]\times$$
$$\times\frac{\partial^3 c}{\partial t\partial x^2}+\frac{\tau^* u(2B-A)}{\beta(A+B)}\frac{\partial^3 c}{\partial t^2\partial x}+\frac{AB\tau^*}{\beta(A+B)}\frac{\partial^3 c}{\partial t^3}+\frac{u}{\beta}\left[D_V-\frac{\tau^* u^2}{A(A+B)}\right]\frac{\partial^3 c}{\partial x^3}=0. \tag{2}$$

It has been established in [9] that Eq. (2) yields wave solutions and generally determines three different density waves. It is readily seen that, because of its complexity, (2) is not suitable for practical application. We shall, therefore, attempt to derive from it a simpler equation which would make it possible to simulate vertical disperse material and heat transport in a fluidized bed.

It was shown in [10] that, for $u_1 = u = 0$; $\tau^* = 0$, system (1) has solutions which rapidly degenerate with respect to the parameter β (actually, for $\beta H^2/D_V \geq 10$, they no longer depend on $\beta H^2/D_V$ and are transformed into the solutions of the classical diffusion equation). As a rule, $\beta H^2/D_V > 10$ in fluidized-bed apparatus [10]. This fact made it possible to describe in [9, 11] the horizontal mixing of particles and heat transfer in the bed by a hyperbolic second-order diffusion equation (obtained from (2) for $u_1 = u = 0$, by substituting D_h for D_V), rather than by a third-order equation of the type (2) for $u_1 = u = 0$; $\beta \rightarrow \infty$. Considering this, we shall attempt to derive from (2) a certain second-order equation for describing the vertical movement of particles, using as a basis the behavior of (2) for $\beta \rightarrow \infty$.

The condition $\beta \rightarrow \infty$ in (2) lowers its order and leads to a second-order equation:

$$\frac{\partial c}{\partial t}+\tau^*\frac{\partial^2 c}{\partial t^2}+\tau^* u_1\frac{\partial^2 c}{\partial t\partial x}=D_V\frac{\partial^2 c}{\partial x^2}. \tag{3}$$

An analysis of this equation based on the method of characteristics is given in [9]. Direct utilization of (3) for describing vertical mixing of the disperse material is inadmissible, if only because it does not contain the axial "Taylorian" diffusion of particles with the coefficient $u^2/\beta(A + B)$ (see the coefficient in front of d^2c/dx^2 in (2)). This diffusion mechanism can be substantial in systems with intensive circulation flow. Therefore, our aim here is to derive an equation similar to (3) that would account for the "Taylorian" particle diffusion. The validity of the assumptions made will then be checked by comparing the theoretical and the experimental diffusion curves for heated particles.

Consider the following equation:

$$\frac{\partial c}{\partial t}+\tau^*\left[1+\frac{u^2}{\beta(A+B)D_V}\right]\left(\frac{\partial^2 c}{\partial t^2}+u_1\frac{\partial^2 c}{\partial t\partial x}\right)=\left[D_V+\frac{u^2}{\beta(A+B)}\right]\frac{\partial^2 c}{\partial x^2}. \tag{4}$$

It satisfies the rational requirements for $\beta \to \infty$, $t \to \infty$, $\tau^* \to 0$, accounts for the axial "Taylorian" diffusion of particles, and correctly describes the velocities of the forward (w_f) and reverse (w_r) waves in the descending emulsion phase (see [9]). It is true that (4) does not contain the third wave with the velocity u_2 (the velocity of bubble trails). However, considering the smallness of B (in comparison with A), which is usual for a fluidized bed, we can assume that the above fact is of no great consequence, all the more so as the balance ascent of particles in gas bubble trails is taken into account in (4).

Let us explore the possibility of using Eq. (4) to describe the vertical transport of heat and solid particles in a fluidized bed. Consider the following boundary-value problem for (4), written for the case of vertical heat transport ($c \to T$, $\beta \to \alpha$, $D_v \to a_v$):

$$\frac{\partial T}{\partial t} + \varphi\tau^* \left(\frac{\partial^2 T}{\partial t^2} + u_1 \frac{\partial^2 T}{\partial t \partial x}\right) = \left[a_v + \frac{u^2}{\alpha (A+B)}\right] \frac{\partial^2 T}{\partial x^2}, \quad (5)$$

$$T(0,\ x) = \begin{cases} T_k, & 0 \leqslant x < h, \\ \dfrac{T_k + T_0}{2}, & x = h, \\ T_0, & h < x \leqslant H, \end{cases} \quad \frac{\partial T(0,\ x)}{\partial t} = 0; \quad (6)$$

$$J = -\varphi a_v \frac{\partial T}{\partial x} - \varphi\tau^* \left(\frac{\partial J}{\partial t} + u_1 \frac{\partial J}{\partial x}\right) = 0, \quad x = 0;\ H.$$

We shall use the following auxiliary system of equations for solving the stated problem, which simulates the actual experimental conditions with regard to critical heat transport in a fluidized bed on the basis of the two-dimensional thermal pulse method (see, for instance, [12]):

$$A^0 \frac{\partial T_1}{\partial t} + A^0 w_f \frac{\partial T_1}{\partial x} = \alpha^0 (T_2 - T_1),$$
$$B^0 \frac{\partial T_2}{\partial t} - B^0 w_0 \frac{\partial T_2}{\partial x} = \alpha^0 (T_1 - T_2), \quad (7)$$

with the conditions $A^0 + B^0 = A + B$; $A^0 w_v = B^0 w_0 = u^0$; $w_f = \sqrt{\frac{a_v}{\tau^*} + \frac{1}{4} u_1^2} + \frac{1}{2} u_1$; $w_0 = \sqrt{\frac{a_v}{\tau^*} + \frac{1}{4} u_1^2} - \frac{1}{2} u_1$ (w_f, w_r are the forward and the reverse wave velocities, respectively, which are defined by (5)). It can be shown that we obtain the following from (7) for determining $T = \frac{A^0 T_1 + B^0 T_2}{A^0 + B^0}$:

$$\frac{\partial T}{\partial t} + \frac{A^0 B^0}{\alpha^0 (A+B)} \left(\frac{\partial^2 T}{\partial t^2} + u_1 \frac{\partial^2 T}{\partial t \partial x}\right) = \frac{A^0 B^0 a_v}{\tau^* \alpha^0 (A+B)} \frac{\partial^2 T}{\partial x^2}. \quad (8)$$

The coefficients A^0, B^0, and α^0 can readily be determined so that Eq. (8) corresponds exactly to (5). We use the condition $A^0 B^0 / \alpha^0 (A+B) = \varphi\tau^*$. This condition, together with the one written earlier, $A^0 + B^0 = A + B$; $A^0 w_f = B^0 w_0$, determines A^0, B^0, α^0 unambiguously:

$$\alpha^0 = \frac{(A+B) w_0 w_f}{4\varphi\tau^* \left(\frac{a_v}{\tau_*} + \frac{1}{4} u_1^2\right)}, \quad A^0 = (A+B) w_0 / 2\sqrt{\frac{a_v}{\tau^*} + \frac{1}{4} u_1^2}, \quad (9)$$
$$B^0 = (A+B) w_f / 2\sqrt{\frac{a_v}{\tau^*} + \frac{1}{4} u_1^2}$$

and transform (8) into Eq. (5).

For system (7) with the boundary conditions

$$T_1(0,\ x) = T_2(0,\ x) = \begin{cases} T_k, & 0 \leqslant x < h, \\ (T_k + T_0)/2, & x = h, \\ T_0, & h < x \leqslant H, \end{cases} \quad (10)$$

$$T_1(t,\ 0) = T_2(t,\ 0); \quad T_1(t,\ H) = T_2(t,\ H),$$

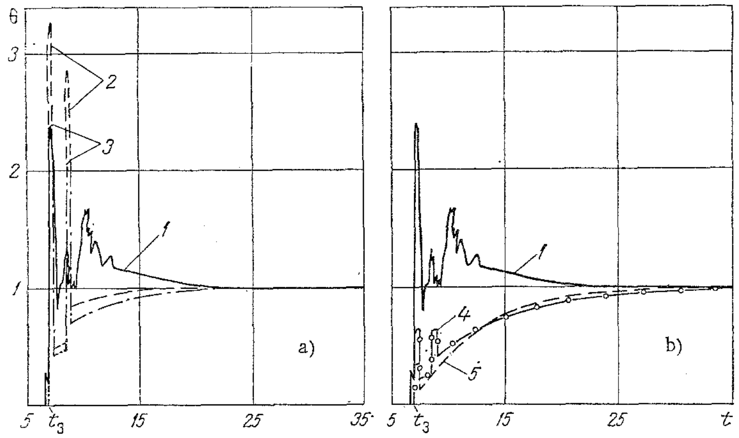

Fig. 1. Comparison between the experimental response function and the theoretical functions based on the hyperbolic model (ξ = 0.92; R = 0.01: 1) experimental function; 2) E = a_V + u^2/α (A+B) = 52 cm²/sec; α = 0.06 1/sec; 3) E = 45 cm²/sec; α = 0.09 1/sec; 4) E = a_V = 32 cm²/sec; α = ∞; 5) E = 45 cm²/sec; α_{eff} = 0.61 1/sec (Van Deemter's circulation model [7]). Quartz sand, d = 0.60 mm; u_0 = 20 cm/sec; u_{fi} = 52 cm/sec; H = 51 cm; checkerboard bunch of horizontal pipes spaced at 60 mm vertically and horizontally; w_f = 6.6 cm/sec; u_1 = 1.2 cm/sec; τ^* = 0.89 sec; t is given in seconds.

a solution has been obtained in [13], which has the following form for the mean temperature T, averaged with respect to the two phases:

$$\overline{\theta}=\frac{T-T_0}{T_h-T_0}=R+\exp(-J_0/\mathrm{Pe})\sum_{n=1}^{\infty}\frac{(-1)^n\sin n\pi R}{n\pi}\left(\Gamma\,\mathrm{ch}\frac{J_0}{\mathrm{Pe}}A_n+\frac{D}{A_n}\,\mathrm{sh}\frac{J_0}{\mathrm{Pe}}A_n\right), \tag{11}$$

where

$$\Gamma=2\cos n\pi(1-\xi);\quad D=\Gamma+2\frac{B^0-A^0}{A+B}\,\mathrm{Pe}\,n\pi\sin n\pi(1-\xi);$$

$$A_n=\sqrt{1-\mathrm{Pe}^2n^2\pi^2};\quad J_0=\frac{A^0-B^0}{A+B}\left(-\xi+R+\frac{2\tau\,\mathrm{Pe}}{A^0-B^0}\right);$$

$$\mathrm{Pe}=u^0/\alpha^0H;\quad \tau=\alpha^0t.$$

With an allowance for the equality $A^0w_f = B^0w_0 = u^0$, the boundary conditions in (10) stipulate the absence of a heat flux at the sections x = 0; H. Therefore, Eq. (7) with boundary conditions (10) corresponds with an accuracy to specifications to system (5)-(6), the solution of which can therefore be obtained from (11) with an allowance for Eq. (9). The thus found solution of the stated boundary-value problem (5)-(6) has the following form:

$$\overline{\theta}=R+\exp\left(-\frac{\frac{2\,\mathrm{Fo}}{\mathrm{Pe}^*}+\xi-R}{\mathrm{Pe}^*+\frac{4\,\mathrm{Fo}^*}{\mathrm{Pe}^*}}\right)\sum_{n=1}^{\infty}\frac{(-1)^n\sin n\pi R}{n\pi}\times$$

$$\times\left(2\cos n\pi(1-\xi)\,\mathrm{ch}\left[\frac{\frac{2\,\mathrm{Fo}}{\mathrm{Pe}^*}+\xi-R}{\mathrm{Pe}^*+\frac{4\,\mathrm{Fo}^*}{\mathrm{Pe}^*}}\sqrt{1-(\mathrm{Pe}^{*2}+4\,\mathrm{Fo}^*)\,n^2\pi^2}\right]+\right. \tag{12}$$

$$\left.+\frac{2\cos n\pi(1-\xi)+2\,\mathrm{Pe}^*\,n\pi\sin n\pi(1-\xi)}{\sqrt{1-(\mathrm{Pe}^{*2}+4\,\mathrm{Fo}^*)\,n^2\pi^2}}\,\mathrm{sh}\left[\frac{\frac{2\,\mathrm{Fo}}{\mathrm{Pe}^*}+\xi-R}{\mathrm{Pe}^*+\frac{4\,\mathrm{Fo}^*}{\mathrm{Pe}^*}}\sqrt{1-(\mathrm{Pe}^{*2}+4\,\mathrm{Fo}^*)\,n^2\pi^2}\right]\right),$$

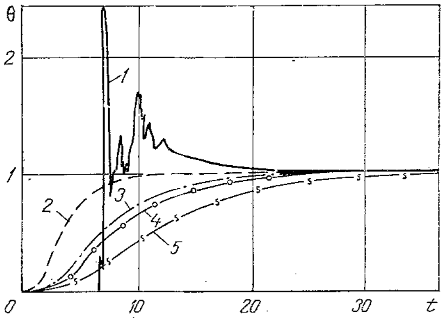

Fig. 2. Comparison between the experimental response function and the theoretical response functions based on the parabolic model (ξ = 0.92; R = 0.01): 1) experimental function (see Fig. 1); 2) E = 104 cm^2/sec; 3) 50; 4) 45; 5) E = 32 cm^2/sec.

where

$$\mathrm{Pe}^* = \frac{\varphi\tau^* u_1}{H}; \quad \mathrm{Fo}^* = \frac{\varphi^2\tau^* a_v}{H^2}; \quad \mathrm{Fo} = \frac{\varphi t a_v}{H^2}.$$

The experimental curves of vertical diffusion of heated particles obtained in [12] were compared with those calculated by means of a computer on the basis of (12). Some typical results of the comparison are shown in Fig. 1. The coefficient of vertical thermal diffusivity of the bed a_v was found with respect to the related coefficient of horizontal thermal diffusivity a_h (see Fig. 3 in [11]) by means of the method described in [9]. The relaxation time τ^* was calculated by using the processing of experiments on horizontal heat transport [9, 11] on the basis of the expression $\tau^* = a_h/v^2$. The velocity of the descending circulatory motion of particles u_1 = 1.2 cm/sec was determined by using the expression for the wave velocity $w_f = \sqrt{\frac{a_v}{\tau^*} + \frac{1}{4}u_1^2} + \frac{1}{2}u_1$ (the value of w_f is readily determined [9] with respect to the lag time (t_3) of the experimental response function (Fig. 1)). The condition for the best agreement between the experimental curve and those calculated by means of (12), which, for the sake of simplicity, was estimated with respect to the height of the first "peak" of the experimental response curve, provided the sought values of the interphase exchange coefficient and, thus, of the effective coefficient of vertical thermal diffusivity (curve 3 in Fig. 1a for α = 0.09 1/sec; E = 45 cm^2/sec). For comparison, Fig. 2 shows the results obtained in calculating the response function by using the solution of the parabolic equation of thermal conductivity:

$$\bar{\theta} = R + 2\sum_{n=1}^{\infty} \frac{\sin n\pi R \cos n\pi\xi}{n\pi} \exp(-n^2\pi^2 \mathrm{Fo}), \tag{13}$$

which is obtained from (12) by limit passage to Fo* = Pe* = 0 (τ^* = 0; u_1 = 0). It is evident that the hyperbolic equation of thermal conductivity provides a significantly better description of the experimental relationships (see Fig. 1) than the parabolic equation (Fig. 2). An important fact should be noted: The coefficient of vertical dispersion of heated particles (the effective coefficient of vertical thermal diffusivity of the bed) E = 50 cm^2/sec, found by means of the classical (parabolic) equation of thermal conductivity and the well-known two-dimensional thermal pulse method [14], proved to be very close to $a_v + u^2/\alpha(A+B)$ = 45 cm^2/sec (for α = 0.09 1/sec). This agrees with the conclusion reached in [11]: The horizontal thermal diffusivity coefficients determined by means of the hyperbolic and the parabolic equations of thermal conductivity were definitely not different from each other.

Figure 1b also provides a comparison between the experimental curve of vertical heat transport and the curve based on solution (11) for Van Deemter's circulation model [7], whose equations are obtained from (7) by using the substitution $A^0 \to A_0$; $B^0 \to B_0$; $w_f \to (u_1)_{eff}$; $w_0 \to u_2$; $\alpha^0 \to \alpha_{eff}$ ($A_0(u_1)_{eff} = B_0u_2$; $A_0 + B_0 = A + B$).† The value of $(u_1)_{eff}$ was w_f = 6.6 cm/sec, while the bubble (trail) velocity u_2 was assumed to be 100 cm/sec. The coefficient α_{eff} was determined from the condition $E = A_0^2(u_1)_{eff}^2/\alpha_{eff}(A+B)$ = 45 cm^2/sec, which actually reduces the effective coefficient of vertical thermal diffusivity of the bed to the axial coefficient of "Taylorian" diffusion of heated particles. For the given specific case, α_{eff} = 0.61 1/sec. It is evident from Fig. 1 that the circulation model of vertical mixing, while describing the initial time lag t_l, renders the subsequent behavior

† It should be noted that the response functions of the circulation model can also be calculated by means of (12), where $\mathrm{Pe}^* = \frac{[(u_1)_{eff} - u_2]\tau^*}{H}$; $\mathrm{Fo}^* = \frac{\tau^* E}{H^2}$; $\mathrm{Fo} = \frac{tE}{H^2}$; $\tau^* = \frac{A_0 B_0}{\alpha_{eff}(A+B)}$; $E = A_0^2 \times (u_1)_{eff}^2/\alpha_{eff}(A+B)$.

of the thermal curve much less adequately than model (5).†

In conclusion, we shall make a few remarks concerning the comparison between the experimental and theoretical mixing curves for heated particles. The statistical nature of the process manifests itself most strongly in vertical mixing, which results in a certain (often considerable) difference between the response functions obtained under identical operating conditions, but at different times. However, the basic features of these functions — existence of the characteristic time lag t_l and the accompanying sharp temperature jump (Figs. 1 and 2) — remain unchanged. The proposed model describes primarily these characteristics in an adequate manner.

In contrast to parabolic equations of diffusion and thermal conductivity, the solutions of a hyperbolic equation of the type (4) or (5) reflect fairly accurately the form of the initial temperature (or density) distribution. It is clear that the initial conditions prevailing in experiments can differ to a certain extent from those assigned in (6). This must be taken into account in analyzing the results of comparison between the experimental and the theoretical curves.

NOTATION

a_v and a_h, coefficients of the vertical and the horizontal thermal diffusivity of the bed, respectively; A, portion of the bed volume occupied by the descending continuous phase (phase A); B, portion of the bed volume occupied by bubble trails (phase B); $A + B = 1 - \varepsilon_b$; C_s, specific heat of solid particles; c_1 and c_2, densities of labeled particles in phases A and B, respectively; d, particle diameter; D_{xx}, element of the coefficient tensor of turbulent particle diffusion in phase A; $D_v = AD_{xx}/(A+B)$, coefficient of vertical diffusion of particles; D_h, coefficient of horizontal diffusion of particles; $E = a_v + u_2/\alpha\ (A+B)$, effective coefficient of vertical thermal diffusivity of the bed; h, width of the heat pulse; H, bed height, $J = Q/\rho C_s$; Q, thermal flux density; $R = h/H$; t, time; T_1 and T_2, particle temperatures in phases A and B, respectively; u_1 and u_2, velocities of the descending emulsion phase and of the bubble trails, respectively; $Au_1 = Bu_2 = u$, circulation velocity of particles with respect to the entire cross section of the apparatus; u_{fi}, filtration rate; u_0, incipient fluidization rate; v, horizontal wave velocity, w_f and w_0, velocities of the forward and reverse waves defined by (5), respectively; x, vertical coordinate; α and β, interphase exchange coefficients; ε_b, bubble density in the bed; $\xi = x/H$; ρ, fluidized bed density; τ^*, relaxation time; $\varphi = 1 + u^2/\alpha(A+B)a_v$; $\theta = \bar{\theta}/R$.

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† It should be mentioned that Van Deemter's circulation model coincides formally with system (7) and corresponds to a second-order hyperbolic equation of the type (3) (with the substitution $u_1 \to (u_1)_{eff} - u_2$). Thus, it constitutes a convenient mathematical scheme of vertical transport of particles, which has been utilized, for instance, in [5, 15]. Without reflecting the actual mixing mechanism, i.e., without being a physically adequate model (the turbulent diffusion of particles in the emulsion phase is missing), this scheme can also describe formally and more or less correctly the actual mixing curves (see Fig. 1). The circulation model, however, is based on the effective values A_0, B_0, $(u_1)_{eff}$, and α_{eff} which are far from the "actual" values of A, B, u_1, and α. This situation must always be kept in mind in analyzing experimental data on the basis of a physically inadequate model.

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CALCULATION OF THE EFFECTIVE PERMITTIVITY OF A TWO-PHASE STREAM

A. P. Vasil'ev

UDC 532.529.5

A calculating equation is proposed for the effective permittivity of bubbly and gas–drop streams.

The methods of electrical conductivity and inductance [1], which are widespread in the diagnostics of two-phase flows, cannot be used to measure the volumetric content of the gaseous phase in a stream of dielectric liquid. In this connection one can use a capacitive method based on measuring the capacitance of a capacitor placed in the two-phase stream.

The dependence of the effective permittivity of a two-phase stream on the volumetric content of the disperse phase in it will be decisive for the use of this method. This dependence must also be at hand in many calculations of electron–ion technology and in problems of the electrohydrodynamics of two-phase flows [2, 3].

We note that methods are known in the literature [4-6] for calculating the coefficients of effective conductivity of heterogeneous (nonflowing) media. Unfortunately, they ignore the possibility of reorganization of the structure of a two-phase stream with an increase in the volumetric content of the disperse phase. For example, the change in the mode of flow of a bubbly stream has a crisis character, so that the coefficients of conductivity should undergo a discontinuity at some limiting attainable volumetric bubble content.

Let us consider a disperse stream of two dielectric media. Let the disperse phase be present in the form of equal-sized spherical drops or bubbles and be characterized by a permittivity ε_2, while the carrier (dispersion) phase is characterized by a permittivity ε_1. We assume that the fluctuations in volumetric content, number density, and sizes of the disperse particles caused by turbulent pulsations and processes of particle fragmentation and coalescence do not exceed their average values φ, N, and R by many times.

Let a small plane capacitor be placed in the two-phase stream so that the functions φ, N, and R can be taken as uniform in the space between its plates. At the same time, the volume of the capacitor is representative, i.e., $a\mathrm{bh} \gg R^3$, so that the nonuniform electrostatic field due to the disperse particles can be averaged.

Let the distance a between the capacitor plates be a multiple of 2R. We divide up the region of the two-phase stream in it into layers of thickness 2R by equipotential planes. In the resulting system of $a/2R$ series-

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